Homework 6 CSE 310 Fall 2008 Arizona State University

Due by Tuesday 10/14 at 9:00

1. What is an optimal Huffman code for the following set of frequencies, based on the first 8 Fibonacci numbers?

$$a:1 \quad b:1 \quad c:2 \quad d:3 \quad e:5 \quad f:8 \quad g:13 \quad h:21$$

   Can you generalize your answer to find the optimal code when the frequencies are the first n Fibonacci numbers?

2. With an adjacency matrix is representation, most graph algorithms require time $\Omega(V^2)$, buth there are some exceptions. Show that determining whether a directed graph contains a **universal sink**—a vertex with indegree $V - 1$ and outdegree 0—can be determined in time $O(V)$, given an adjacency matrix for G.

3. Give an example of a directed graph $G = (V, E)$, a source vertex $s \in V$ and a set of tree edges $E_\pi \subseteq E$ such that for each vertex $v \in V$, the unique path in the graph (V, E_π) from s to v is a shortest path in G, yet the set of edges E_π cannot be produced by running BFS on G, no matter how the vertices are ordered in each adjacency list.

4. The **diameter** of a tree $T = (V, E)$ is given by $\max_{u,v \in V} \delta(u, v)$, that is, the diameter is the largest of all shortest-path distances in the tree. Give an efficient algorithm to compute the diameter of a tree, and analyze the running time of your algorithm.

5. After running DFS, show that edge (u, v) is
   - a tree edge of forward edge if and only if $d[u] < d[v] < f[v] < f[u]$,
   - a back edge if and only if $d[v] < d[u] < f[u] < f[v]$, and
   - a cross edge if and only if $d[v] < f[v] < d[u] < f[u]$.

6. Prove or disprove: if a directed graph G contains cycles, then TOPOLOGICAL-SORT(G) produces a vertex ordering that minimizes the number of "bad" edges that are inconsistent with the ordering produced.