Due by Tuesday 10/21 at 9:00

1. Let uv be a mminimum-weight edge in a graph G. Show that uv belongs to some minimum spanning tree of G.
2. Show that if an edge uv is contained in some minimum spanning tree, then it is a light edge crossing some cut of the graph.
3. Show that a graph has a unique minimum spanning tree if, for every cut of the graph, there is a unique light edge crossing the cut. Show that the converse is not true by giving a counterexample.
4. Argue that if all edge weights of a graph are positive, then any subset of edges that connects all vertices and has minimum total weight must be a tree. Give an example to show that the same conclusion does not follow if we allow some weights to be nonpositive.
5. Suppose G is represented as an adjacency matrix. Give a simple implementation of Prim's algorithm that runs in $O(n^2)$ time.
6.